Effects of finite arm-length of LISA on analysis of gravitational waves from MBH binaries

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Response of an interferometer becomes complicated for gravitational wave shorter than the arm-length of the detector, as nature of wave appears strongly. We have studied how parameter estimation for merging massive black hole binaries are affected by this complicated effect in the case of LISA. It is shown that three dimensional positions of some binaries might be determined much better than the past estimations that use the long wave approximation. For equal mass binaries this improvement is most prominent at $\sim 10^5 M_\odot$.

I. INTRODUCTION

The *Laser Interferometer Space Antenna* (LISA), a joint project of NASA and ESA, is planed to be launched in 2011 [1]. LISA is sensitive to low frequency gravitational waves at 10^{-4}Hz $\lesssim f \lesssim 10^{-1}$Hz, and would directly confirm gravitational radiation from some known Galactic binaries [2]. Thousands of closed white dwarf binaries would be also detected by LISA. They can be regarded as insurances of the project, but other exciting phenomena might be observed [2]. Coalescence of a massive black hole (MBH) binary is the most spectacular and energetic event. Gravitational wave from a MBH binary is in itself very interesting for studies of general relativity, but would also bring significant impacts on astrophysics and cosmology [2], even though event rate is highly unknown at present [3]. Parameter estimation errors are basic measures for discussion of gravitational wave astronomy. For example, with small error box for the position of a MBH binary we might specify its host galaxy and investigate it with various observational tools.

LISA is constituted by three space crafts that keep a triangle configuration with side (arm-length) $l = 5.0 \times 10^6$km and trail $\sim 20^\circ$ behind the Earth [2]. Response of a detector becomes complicated for gravitational waves with wave-length $\lambda \lesssim l$, as the nature of wave appears strongly [4,5,6]. This interesting feature might be an advantage for signal analysis but has not been taken in properly so far. In this paper we discuss how the parameter estimation errors are changed by this effect.

This paper is organized as follows. In Section II we study the effect of a finite arm-length on the phase shift $\delta\phi$ of the detector, and introduce the past analysis that is based on the simple picture using the variations of the arm-lengths δl under the long wave approximation. In Sec III A gravitational wave forms from MBH binaries are briefly discussed. Noise spectrum for both the variations δl and the phase sifts $\delta\phi$ are presented in Sec III B. The spectrum for the latter is simply reproduced from that for the former. In Sec IV our numerical results are shown, and signal to noise ratio and the parameter estimation errors are compared for the two methods (with and without the long wave approximation). Though we mainly study gravitational waves from MBHs, results for nearly monochromatic sources (from *e.g.* Galactic binaries) are also presented. Sec V is devoted to discussion. In Appendix A we make same kind of comparisons for the noise canceling combination of data streams. In Appendix B we present explicit expressions for detector's response to gravitational wave from a binary.

II. DETECTORS WITH FINITE ARM-LENGTH

We name the three vertexes (space crafts) of LISA as A, B and C. Cutler [7] studied parameter estimation of binaries from variations of arm-lengths in the following forms

$$\delta l_{AB}(t) - \delta l_{AC}(t), \quad \{(\delta l_{BA}(t) - \delta l_{BC}(t)) - (\delta l_{CB}(t) - \delta l_{CA}(t))\}/\sqrt{3}. \tag{1}$$

The normalization factor $1/\sqrt{3}$ in the second expression is explained later. Here δl_{XY} is the variation of the arm-length between two space crafts X and Y $(X, Y \in \{A, B, C\},\ X \neq Y)$. This simple picture for detector's response is valid only at long wave limit where gravitational wave-length is much larger than the detector's arm-length (see also [8,9]). With quantities more close to detector's output [2] these two data should be replaced by

$$\delta\phi_{AB}(t) - \delta\phi_{AC}(t), \quad \{(\delta\phi_{BA}(t) - \delta\phi_{BC}(t)) - (\delta\phi_{CB}(t) - \delta\phi_{CA}(t))\}/\sqrt{3}, \tag{2}$$

where $\delta\phi_{XY}$ represents the single-arm phase shift of the laser beam that leaves a detector (vertex of the triangle) X for Y and then returns back to X. The quantity $\delta\phi_{AB} - \delta\phi_{AC}$ is an interferometer signal obtained at the detector A.

In Ref. [7] it is assumed that the noise for the time variations δl_{XY} is stationary, Gaussian and symmetric among three arms. We make same assumption for the corresponding phase shifts $\delta\phi_{XY}$ In this case noises of the above two data (2) (or (1)) do not correlate to each other.

Response of the phase shift $\delta\phi_{AB}$ for gravitational wave h incoming from a direction $\boldsymbol{\Omega}_s$ is expressed as follows [4,5,10]

$$\frac{1}{2\pi\nu_0}\frac{d\delta\phi_{AB}(t)}{dt}=\frac{1}{2}\cos 2\psi_{AB}[(1-\cos\theta_{AB})h(t,A)+2\cos\theta_{AB}h(t-\tau,B)-(1+\cos\theta_{AB})h(t-2\tau,A)], \tag{3}$$

where θ_{AB} is the angle between the source direction $\boldsymbol{\Omega}_s$ and the arm $\boldsymbol{x}_A-\boldsymbol{x}_B$, and ν_0 is the fundamental frequency of the laser. ψ_{AB} is the principle polarization angle of the quadrupole gravitational wave h for the arm $\boldsymbol{x}_A-\boldsymbol{x}_B$. We denote the polarization basis tensor H_{ab} of the wave as $H_{ab}=p_ap_b-q_aq_b$ using two orthogonal vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ with $\boldsymbol{p}\cdot\boldsymbol{\Omega}_s=\boldsymbol{q}\cdot\boldsymbol{\Omega}_s=\boldsymbol{p}\cdot\boldsymbol{q}=0$ and $|\boldsymbol{p}|=|\boldsymbol{q}|=1$. Then the angle ψ_{AB} is given by $\tan\psi_{AB}=(\boldsymbol{q}\cdot(\boldsymbol{x}_A-\boldsymbol{x}_B))/(\boldsymbol{p}\cdot(\boldsymbol{x}_A-\boldsymbol{x}_B))$ [10]. The quantity $h(t,A)$ is value of the gravitational wave at point A and time t, and can be expressed as $h(t+\boldsymbol{\Omega}_s\cdot\boldsymbol{x}_A(t)/c)$ in the present case. We have denoted the propagation time of light for the arm l by $\tau\equiv l/c$, and neglected very small relative motions between the vertexes in the time scale τ. In Appendix B we present explicit expression of Eq.(3) for a binary source.

When the gravitational wave-length is much larger than the arm-length l, namely $f\ll(2\pi\tau)^{-1}\equiv f_{arm}$, structure of wave is irrelevant at the spatial scale l and the wave h causes variations $\delta\phi_{AB}$ or δl_{AB} in very simple manners. In this long wave limit the phase shift is directly proportional to the variation of the arm-length δl_{XY} as $\delta\phi_{XY}/2\pi\nu_0=\delta l_{XY}/2c$, and two data sets (1) and (2) are equivalent. We have the following relation by perturbative evaluation of Eq.(3) (see also Ref. [10])

$$\frac{\delta\phi_{AB}}{2\pi\nu_0\tau}=\sin^2\theta_{AB}\cos 2\psi h(t)=\frac{\delta l_{AB}}{2l}. \tag{4}$$

LISA has arm-length $l=5.0\times10^6$ km, corresponding to the critical frequency $f_{arm}=0.01$Hz. Note that the frequency f does not appear in the above relation (4). In this long wave limit the angular pattern function (usually denoted as $F^{+,\times}$ [11]) contains complete information of the angular dependence of the interferometer signal (*e.g.* $\delta\phi_{AB}-\delta\phi_{AC}$). Two data streams (1) are equivalent to responses of two 90°-interferometers rotated by 45° (see discussion in [7]). The factor $1/\sqrt{3}$ in the second expression of (1) or (2) is given for this purpose.

When the gravitational wavelength c/f is comparable to or shorter than the arm-length l, the phase shifts $\delta\phi_{XY}$ show complicated response that depends strongly on the frequency f and the source direction relative to the detector (see Appendix B). As discussed earlier, this troublesome but interesting effects have been cut down so far in analysis of parameter estimation errors for binary sources. Past analysis was performed in the following manner [7,8,9].

(i) The angular averaged transfer function $T(f)$ between the phase shift $\delta\phi$ and wave amplitude h was used $\delta\phi\propto T(f)h$ to include amplitude modulation caused by finiteness of the arm-length $l\neq0$. Its effect can be effectively absorbed in the noise curve $\sqrt{S_h(f)}$ for the wave amplitude h.

(ii) Then the simple analysis with the variation of the arm-length δl_{XY} (valid only at the long wave limit) was applied with the angular pattern function $F^{+,\times}$ for the angular dependence of the response (and also Doppler phase caused by velocity of detectors).

At high frequency limit $f\gg f_{arm}$ the transfer function becomes $T(f)\propto f^{-1}$ due to cancellation of wave within arms. The measurement sensitivity of LISA for the phase shift $\delta\phi$ is $\sqrt{S_{\delta\phi}(f)}\propto f^0$ (in units of Hz$^{-1/2}$) at 10^{-2}Hz $\lesssim f\lesssim 10^0$Hz where the shot noise is dominant source of noise. Thus LISA has $\sqrt{S_h(f)}\propto\sqrt{S_{\delta\phi}(f)}T(f)^{-1}\propto f$ at high frequency region [2,4,5].

Here we discuss basic aspects of signal analysis with paying attention to the finiteness of the arm-length l. The most important quantity for detection of gravitational wave is the signal to noise ratio (SNR) [11]. For its estimation the above mentioned method with the angular averaged transfer function would be effective for LISA considering its rotation and frequency average of chirping signals. But parameter estimation errors (*e.g.* error box for direction of a source) might become much smaller than the previous analysis when we include the complicated response of the phase shift as it is. Gravitational wave signal measured by a detector (as expression (2)) is given by values of gravitational wave h at different positions and times. Time delay between them would also affect parameter estimation. These two effects, time delay and cancellation within the arms, are natural outcome of the finiteness of the arm-length and appear in a similar form, namely, linear combination of terms like $\exp[2\pi if\boldsymbol{\Omega}_s\cdot(\boldsymbol{x}_A-\boldsymbol{x}_B)/c]$ in Fourier space of the time variable t. Our primary aim in this paper is to show how the parameter estimation would be changed if we simply replace the arm-length variation δl_{XY} (as in Eq.(1)) with the corresponding phase shifts $\delta\phi_{XY}$ (as in Eq.(2)) without using the long wave approximation.

III. ANALYSIS OF GRAVITATIONAL WAVES FROM MERGING MASSIVE BLACK HOLE BINARIES

A. Gravitational Waveform

We investigate the parameter estimation errors expected in matched filtering method [12]. When the wave signal contains fitting parameters $\{\gamma_i\}$, their estimation errors (variances) for signal analysis are evaluated with using the Fisher's information matrix Γ_{ij} as $\langle \Delta\gamma_i \Delta\gamma_j \rangle = \Gamma_{ij}^{-1}$. We analyze the phase shifts (2) by extending earlier studies to include effects caused by the arm-length, but other points are almost identical to Ref. [7] that uses the variations (1) under the long wave approximation. Detailed analysis for the phase shift is given in Appendix B.

For gravitational wave emitted by a binary we adopt the restricted post-Newtonian approach, but higher-order harmonics could become important in some cases [13]. We use the 1.5PN phase for a circular orbit [14]

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4}(8\pi G c^{-3} \mathcal{M}_z f)^{-5/3} \left[1 + \frac{20}{9} \left(\frac{943}{336} + \frac{11\mu_z}{4M_z} \right) x + (4\beta - 16\pi) x^{3/2} \right], \tag{5}$$

where t_c and ϕ_c are integration constants, β is the spin-orbit coupling term, μ_z, M_z and $\mathcal{M}_z$ are the reduced mass, the total mass and the chirp mass of the binary. All of the mass parameters are multiplied by the factor $(1+z)$ (z: redshift of the binary) with suffix z. The total mass M_z is expressed by other two masses as $M_z = \mathcal{M}_z^{5/2} \mu_z^{-3/2}$. The post-Newtonian expansion parameter x is defined as $x \equiv \left\{ G\pi c^{-3} M_z f \right\}^{2/3}$. We have used the stationary phase approximation for the Fourier transformed waveform. In our analysis total number of the fitting parameters is 10 as $\mathcal{M}_z, \mu_z, \beta, \phi_c, t_c, \ln D$(luminosity distance) and $(\theta_s, \phi_s), (\theta_l, \phi_l)$. The latter four parameters define the direction $\boldsymbol{\Omega}_s$ and the orientation $\boldsymbol{\Omega}_l$ of the binary in a fixed frame at the barycentre of the solar system. In this paper we use the error ellipse

$$\Delta\boldsymbol{\Omega}_i \equiv 2\pi \sin\theta_i \sqrt{\langle \Delta\theta_i^2 \rangle \langle \Delta\phi_i^2 \rangle - \langle \Delta\theta_i \Delta\phi_i \rangle^2}, \tag{6}$$

for the angular parameters ($i = s, l$) [7] and $\Delta V \equiv \Delta D \Delta\boldsymbol{\Omega}_s$ for the three dimensional position error of a source. We also fix $\beta = 0$ for the true value of the parameter β.

Unless stated explicitely, we put the upper cut-off frequency f_{cut} of signal integration at f_{isco} when the binary separation becomes $r = 6M_z/c^2(1+z)$, roughly corresponding to the inner most stable circular orbit. The observed wave frequency at this separation becomes

$$f_{isco} = \frac{6^{-3/2} c^3}{G\pi M_z} = 0.022 \left(\frac{M_z}{2 \times 10^5 M_\odot} \right)^{-1} \text{Hz}. \tag{7}$$

We start integration of the wave signal from the time when the binary is 1yr before its coalescence in the observer's frame. The gravitational wave frequency f_0 at this starting time is given as follows

$$f_0 = 1.9 \times 10^{-4} \left(\frac{\mathcal{M}_z}{0.87 \times 10^5 M_\odot} \right)^{-5/8} \text{Hz}. \tag{8}$$

B. Noise Curve

We make quantitative analysis using noise curve of LISA that is sum of the instrumental noise and the binary confusion noise. For the instrumental noise spectrum given for the wave amplitude h (almost equivalently for δl) we adopt the following function [7]

$$S_i(f) = 5.049 \times 10^5 [\alpha_1(f)^2 + \alpha_2(f)^2 + \alpha_3(f)^2] \text{ Hz}^{-1}, \tag{9}$$

where $\alpha_1(f) = 10^{-22.79}(f/10^{-3}\text{Hz})^{-7/3}$ is mainly the acceleration noise, $\alpha_2(f) = 10^{-23.04}$ is mainly the shot noise, and $\alpha_3(f) = 10^{-24.54}(f/10^{-3}\text{Hz})$ approximately represents the cancellation effects due to the finiteness of the arm-length. The last term reflects the angular averaged transfer function as explained before, and becomes important at $f \gtrsim 10^{-2}$Hz (see *e.g.* [4]). We also include the binary confusion noise $S_c(f)$ (estimated for one year observation) that is stronger than the instrumental one at $f \lesssim 10^{-2.5}$Hz and shows strong dependence on frequency f [15]. Around $f \gtrsim 10^{-2.75}$Hz the confusion noise decreases significantly as the Galactic binaries are resolved at higher frequencies. Its explicite expression is given as follows [7]

$$S_c(f) = \begin{cases} 10^{-42.685} f^{-1.9} \mathrm{Hz}^{-1}, & f \le 10^{-3.15}, \\ 10^{-60.325} f^{-7.5} \mathrm{Hz}^{-1}, & 10^{-3.15} \le f \le 10^{-2.75}, \\ 10^{-46.85} f^{-2.6} \mathrm{Hz}^{-1}, & 10^{-2.75} \le f, \end{cases} \tag{10}$$

where f is written in units of Hz.

Now let us simply recover the noise spectrum $S_{\delta\phi}(f)$ for the phase shift $\delta\phi_{AB} - \delta\phi_{AC}$ from that for the variation δl. At low frequency region $f \ll f_{arm}$ they are simply related to each other as expressions (1) (2) and are essentially equivalent (see Eq.(4)). We use the following functional shape for the instrumental noise of the phase shift $S_{\delta\phi}(f)$ as $S_{\delta\phi}(f) \propto (\alpha_1(f)^2 + \alpha_2(f)^2)$ by removing the effect of the angular averaged transfer function from $S_i(f)$. Note that the confusion noise is expected to be much smaller than the instrumental one at the relevant frequencies $f \gtrsim f_{arm}$. For the second data of expression (2) we use the same noise curve as the first one.

Our calculation using the phase shift is much more complicated than the past one. But our numerical results should coincide with these by past one when we decrease the arm-length l and use the shape of the noise curve $S_h(f)$ instead of $S_{\delta\phi}(f)$. We have confirmed that the parameter errors $\Delta\gamma_i$ given in table 2 of Ref. [7] (obtained by past simple calculation) is reproduced with this procedure.

IV. RESULTS

We have analyzed gravitational waves from MBH binaries with 300 realizations of random directions $\boldsymbol{\Omega}_s$ and orientations $\boldsymbol{\Omega}_l$. Firstly, their SNRs and estimation errors of the fitting parameters $\{\gamma_i\}$ are calculated in both (i) past approach (suffix 0) with angular averaged transfer function under the long wave approximation, and (ii) our method (suffix L) based on the phase shifts of expression (2). In figure 1 we present distribution for ratios of SNRs by two methods $(SNR)_L/(SNR)_0$ and position errors $\Delta V_L/\Delta V_0$, $(\Delta\boldsymbol{\Omega}_s)_L/(\Delta\boldsymbol{\Omega}_s)_0$ and $\Delta D_L/\Delta D_0$ obtained for equal mass MBH binaries with redshifted masses $M_z = 10^4 + 10^4, 10^5 + 10^5$ and $10^6 + 10^6 M_\odot$. Note that these ratios do not depend on cosmological parameters or distance D to the MBH binaries. As shown in the bottom panel of figure 1, ratios of SNRs are close to unity and difference of two methods is very small (within 20%). However, two methods show considerable difference for the position errors. For $10^5 + 10^5 M_\odot$ MBH binaries the errors ΔV_L become typically 10 times smaller than the past estimate ΔV_0. For some samples the errors ΔV are reduced even by a factor $\sim 10^{-2}$. Comparing errors $\Delta\boldsymbol{\Omega}_s$ and ΔD, the former become smaller by the finiteness of the arm-length than the latter. But note that the angular resolution $\Delta\boldsymbol{\Omega}_s$ is, roughly speaking, a product of two errors $\Delta\theta_s$ and $\Delta\phi_s$ as in Eq.(6).

Interestingly enough, difference between two methods is not a monotonic function with respect to BH mass and is most prominent around $\sim 10^5 M_\odot$. This mass dependence can be understood as follows. At higher masses $\gg 10^5 M_\odot$ the upper cut-off frequency f_{isco} is smaller than the critical frequency f_{arm}. Therefore nothing is different between two methods with expressions (1) and (2). With the quadrupole formula for gravitational radiation the time before coalescence is given by the frequency f and the chirp mass $\mathcal{M}_z$ as follows [11]

$$t_{GW} = 8.4 \times 10^4 \left(\frac{f}{10^{-2.75}\mathrm{Hz}} \right)^{-8/3} \left(\frac{\mathcal{M}_z}{0.87 \times 10^5 M_\odot} \right)^{-5/3} \mathrm{sec}. \tag{11}$$

For lower mass BH binaries ($\ll 10^5 M_\odot$) LISA moves longer than the arm-length l during the phase $f \gtrsim 10^{-2.75}$Hz where signal becomes very strong due to decrease of the binary confusion noise. Thus effective baseline of the detector becomes larger than the arm-length $l = 5.0 \times 10^6$km for smaller mass BH binaries and impact of the finite arm-length would decrease.

Now we investigate various aspects of parameter estimation caused by the finiteness of the arm-length using a specific set of binary parameters. We pick up the binary that has the smallest volume ratio $\Delta V_L/\Delta V_0 = 0.011$ in 300 realization of figure 1 for $10^5 + 10^5 M_\odot$ MBH binaries. It has angular parameters $\theta_s = 2.49, \phi_s = 0.03, \theta_l = 2.32$ and $\phi_l = 4.46$. Here we present the estimation errors $\Delta\gamma_i$, not ratios as in figure 1. To normalize the amplitude of the signal we take the redshift of MBH binaries at $z = 1$ with cosmological parameters $\Omega_0 = 0.3, \lambda_0 = 0.7$ and $H_0 = 75$km/sec/Mpc. In figure 2 estimation errors $\Delta\boldsymbol{\Omega}_s, \Delta\boldsymbol{\Omega}_l, \Delta D/D$ and $\Delta\mu_z/\mu_z$ are presented as functions of the upper cut-off frequency f_{cut} that was fixed at f_{isco} with Eq.(7) in the case of figure 1. We fix the lower cut-off frequency at f_0 (eq.[8])

We have found that the intrinsic binary parameters such as $\mathcal{M}_z, \mu_z, \beta, t_c$ and ϕ_c depend weakly on the cut-off frequency f_{cut}. However, errors for the binary position $\Delta\boldsymbol{\Omega}_s$, $\Delta D/D$ or its orientation $\Delta\boldsymbol{\Omega}_l$ decrease significantly at $f_{cut} \gtrsim 0.01$Hz where the long wave approximation breaks down. This shows remarkable contrast to the past analysis shown with thin lines. Our results seem reasonable, as the response of a detector with finite arm-length depends strongly on the direction of the source $\boldsymbol{\Omega}_s$, and information of the distance D or orientation $\boldsymbol{\Omega}_l$ is tightly correlated to them (Appendix B). Significant reduction of position errors ΔV_L at higher frequencies would give further motivation

for studies of nonlinear gravitational dynamics (*e.g.* Post-Newtonian approach) that is a very tough problem on general relativity. By analyzing gravitational wave close to the final coalescence we might identify the host galaxy of a MBH binary!

Next let us make hypothetical experiments to clarify some interesting points. We use the same set of the parameters as in figure 2 with f_{cut} given by Eq.(7). As commented before, the finiteness of the arm-length causes two similar effects (i) cancellation of waves within the arm and (ii) time delay between the vertexes of LISA. To extract effects only of the latter we calculate the volume error $\Delta V_{L'}$ using data from three interferometers that exist at three vertexes of LISA but have arm-length $l \to 0$ with angular averaged transfer function for sensitivity of h. Thus only the positions (separation) of the detectors are different from the past analysis that take the separation $l = 0$. We obtain $\Delta V_{L'}/\Delta V_0 = 0.15$. This result indicates that two effects work cooperatively. Next we stop motion of LISA and keep its position at time $t = t_c$. In this case the volume error $\Delta V_{L''}$ becomes $\Delta V_{L''}/\Delta V_0 = 0.012$ and is very close to $\Delta V_L/\Delta V_0 = 0.011$ that includes motion of LISA. In the past analysis (with $l = 0$) we use the amplitude modulation through the pattern function and the Doppler phase modulation both caused by motion of LISA [2], and cannot solve degeneracy of sources direction $\boldsymbol{\Omega}_s$ and other variables when LISA stops. The response of a detector with $l \neq 0$ depends strongly both on angular variables and frequency of incoming waves at $f \gtrsim f_{arm}$ (see Appendix B). Thus we can solve the degeneracy for chirping binaries even without motion of LISA, though there would be two solution for detector's signal due to the symmetry of source-detector configuration. This is a qualitatively interesting point.

Considering the mass dependence of our results, it is expected that the past simple method using expression (1) under the long wave approximation would be effective for studying galactic compact binaries. These binaries with $f \lesssim 0.1$Hz are nearly monochromatic and have long durations $t_{GW} \gg 1$yr in the LISA band due to their small chirp masses [2]. We have investigated binaries with $m_1 = m_2 = 1M_\odot$, and the time to coalescence $t_{GW} = 30$yr and 100yr. The wave frequency becomes $f = 0.071$Hz at $t_{GW} = 30$yr and $f = 0.045$Hz at $t_{GW} = 100$yr. We fix the observation period at 1yr, thus do not observe the final coalescence in contrast to the analysis for MBH binaries. We calculate parameter estimation errors in this situation with random direction $\boldsymbol{\Omega}_s$ and orientation $\boldsymbol{\Omega}_l$. It is found that the differences between (ΔV_L,ΔV_0) or ($(SNR)_L$,$(SNR)_0$) are less than 15 percent. Recalling that we have made a simple treatment for the effects of the transfer function, this result seems excellent.

V. DISCUSSION

In this paper we have studied how data analysis of gravitational waves from MBH binaries are affected by finiteness of the arm-length of LISA with using the phase shift of detectors. We have numerically confirmed that the past method with the long wave approximation is very effective for estimation of SNRs that are the most important quantities for detection of gravitational waves. However, LISA is able to observe merging MBH binaries with significant SNRs ($\gtrsim 1000$) even at cosmological distances [2,7,8,9], and aspects of gravitational wave astronomy are more relevant for them, rather than SNRs. We have examined the parameter estimation errors $\Delta\gamma_i$ expected in both two methods, and shown that three dimensional position of MBH binaries could be determined much better than the past estimations. In the case of equal mass binaries differences between two methods are most prominent at $\sim 10^5 M_\odot$ and the volume of the error box can decrease significantly. We have shown that (i) this reduction is mainly cause by gravitational waves close to the final coalescence and (ii) position of chirping binaries can be in principle, estimated even without motion of LISA. The former would give further meaning for studies on strong gravitational dynamics, and the latter makes remarkable contrast to former discussions.

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APPENDIX A: NOISE CANCELING COMBINATION

In the main text we have basically followed the model for signal analysis in Ref. [7], and simply replace the variations (1) with the corresponding phase shifts (2) without resorting to the long wave approximation. It has been discussed recently that the laser phase noise can be removed well by devising combination of data at different times [16]. This noise is caused by un-equal arm-lengths of space detectors in contrast to the ground-based ones. In this appendix we discuss parameter estimation using this combination. Let us consider the situation that the space crafts B and C coherently transmit laser beams back to the space craft A. Then the following data stream $X(t)$ is a noise canceling combination

$$X(t) = \delta\phi_{AB}(t) - \delta\phi_{AC}(t) - \delta\phi_{AB}(t - 2l_{AC}/c) + \delta\phi_{AC}(t - 2l_{AB}/c). \tag{A1}$$

For quantitative study we take the limit $l_{AB} = l_{AC} = l = 5.0 \times 10^6$km. Then the Fourier transformation of $X(t)$ is related to that for $\delta\phi_{AB}(t) - \delta\phi_{AC}(t)$ as

$$X(f) = (1 - \exp(2\pi i f l/c))(\delta\phi_{AB}(f) - \delta\phi_{AC}(f)). \tag{A2}$$

In this case the noise curve for the gravitational wave amplitude with data X is identical to that with $\delta\phi_{AB} - \delta\phi_{AC}$ [4], if we only include the acceleration and shot noises.

In figure 3 we present the two ratios $(SNR)_L/(SNR)_0$ and $\Delta V_L/\Delta V_0$ for the noise canceling combination X given in Eq.(A1). Note that the factor $(1 - \exp(2\pi i l f/c))$ in Eq.(A2) contains none of our ten fitting parameters. Thus results for the data $X(t)$ is identical to that for the single data stream $\delta\phi_{AB} - \delta\phi_{AC}$. Figure 3 shows same kind of mass dependence as figure 1. Difference between ΔV_L and ΔV_0 is most prominent at mass $\sim 10^5 M_\odot$ again. But the ratio $\Delta V_L/\Delta V_0$ is generally close to unity and effects of the finite arm-length are smaller.

Note added After submitting this paper, there appears [17] that discusses three data streams A, E, T whose noises do not correlate. Our study can be easily extend to these data and more realistic results would be obtained.

APPENDIX B: PHASE SHIFT OF A DETECTOR FOR GRAVITATIONAL WAVE FROM BINARIES

In this appendix we give an explicit expression for the phase shift in the form (eq.[3])

$$\frac{1}{2\pi\nu_0}\frac{d\delta\phi_{AB}(t)}{dt}. \tag{B1}$$

The shift $\delta\phi_{AB}$ is defined for the laser beam that leaves the detector A for B, and then returns back to A. For source of gravitational radiation h we consider a chirping binary at luminosity distance D, direction $\boldsymbol{\Omega}_s$, and orientation $\boldsymbol{\Omega}_l$. We denote $\boldsymbol{x}_Y$ as the position vector of a detector $Y \in \{A, B, C\}$, and use a coordinate system fixed to the barycentre of the solar system. As we do not directly use the angular pattern functions F_+ and $F_\times$ (see [11]), there is no need to introduce a coordinate system fixed to detectors.

The inclination angle i of the binary is given as

$$\cos i = \boldsymbol{\Omega}_s \cdot \boldsymbol{\Omega}_l. \tag{B2}$$

We denote the propagation time of light for the arm-length by $\tau \equiv |\boldsymbol{x}_A - \boldsymbol{x}_B|/c$, and define the angle θ_{AB} between the direction of a binary $\boldsymbol{\Omega}_s$ and the arm $\boldsymbol{x}_A - \boldsymbol{x}_B$ as

$$\cos\theta_{AB} = \frac{\boldsymbol{\Omega}_s \cdot (\boldsymbol{x}_A - \boldsymbol{x}_B)}{c\tau}. \tag{B3}$$

It is convenient to use the principle axes $(\boldsymbol{p}, \boldsymbol{q})$ for analysis of gravitational wave from a binary. These two vectors are expressed in terms of direction $\boldsymbol{\Omega}_s$ and orientation $\boldsymbol{\Omega}_l$ as

$$\boldsymbol{p} = \frac{\boldsymbol{\Omega}_s \times \boldsymbol{\Omega}_l}{|\boldsymbol{\Omega}_s \times \boldsymbol{\Omega}_l|}, \quad \boldsymbol{q} = -\boldsymbol{\Omega}_s \times \boldsymbol{p}. \tag{B4}$$

With these vectors gravitational wave from a binary is decomposed to two polarization ($+$ and $\times$) modes whose phases differs by $\pi/2$. The plus ($+$) mode has polarization basis tensor $H^+_{ab} = p_a p_b - q_a q_b$ and the amplitude A_+ at the Newtonian order as

$$A_+ = \frac{2G^{5/3}\mathcal{M}_z^{5/3}}{Dc^4}(\pi f)^{2/3}(1 + \cos^2 i), \tag{B5}$$

where $\mathcal{M}_z$ is the redshifted chirp mass of the binary. For this mode the principle polarization angle ψ_{AB} is given by

$$\tan\psi_{AB} = \frac{(\boldsymbol{x}_A - \boldsymbol{x}_B)\cdot\boldsymbol{q}}{(\boldsymbol{x}_A - \boldsymbol{x}_B)\cdot\boldsymbol{p}}. \tag{B6}$$

The cross ($\times$) mode has the amplitude

$$A_\times = \frac{4G^{5/3}\mathcal{M}_z^{5/3}}{Dc^4}(\pi f)^{2/3}\cos i, \tag{B7}$$

with the polarization tensor $H^{\times}_{ab} = p_a q_b + q_a p_b$. Its principle polarization angle differs by $\pi/4$ from the plus mode.

Now we can write down the phase shift (B1) with various parameters of the binary. For notational simplicities we define a function $U(t, Y)$ that contains information of the phase of the gravitational wave at the detector Y and time t (taking its origin appropriately) as

$$U(t, Y) = \exp[2\pi i f(t + \boldsymbol{x}_Y \cdot \boldsymbol{\Omega}_s/c)]. \tag{B8}$$

Then the quantity V is the real part of the following expression (Eq.(3))

$$\begin{aligned}&\frac{1}{2}\left(\cos 2\psi_{AB} A_+ + i \sin 2\psi_{AB} A_\times\right) \\ &\times \left[(1-\cos\theta_{AB})U(t,A) + 2\cos\theta_{AB} U(t-\tau, B) - (1+\cos\theta_{AB})U(t-2\tau, A)\right].\end{aligned} \tag{B9}$$

The expression in the square bracket of the above result can be written as

$$u(t, A) R(\theta_{AB}, f\tau), \tag{B10}$$

where we have defined the factor R as

$$R(\theta_{AB}, f\tau) \equiv \left[(1-\cos\theta_{AB}) + 2\cos\theta_{AB} \exp\left\{2\pi i f\tau(-\cos\theta_{AB} - 1)\right\} - (1+\cos\theta_{AB})\exp(-4\pi i f\tau)\right]. \tag{B11}$$

It is a simple task to obtain the Fourier transformation of the phase shift $\delta\phi_{AB}$. In a similar manner we can make the expression for the phase shift of another arm *e.g.* $\delta\phi_{AC}$. Then we obtain the signals such as $\delta\phi_{AB} - \delta\phi_{AC}$.

When gravitational wave-length is much smaller than the arm-length ($f\tau \gg 1$), the factor R shows complicated response that depends strongly on both the angle $\boldsymbol{\Omega}_s$ and the frequency f. In the long wave approximation ($f\tau \ll 1$) we can perturbatively expand Eq.(B11), and obtain

$$R = 4\pi i f \tau \sin^2\theta_{AB}. \tag{B12}$$

Now the factor R becomes very simple.

Parameters such as $\mathcal{M}_z$, μ_z, β are closely related to the time evolution of the frequency (chirp signal) due to gravitational radiation reaction as given in Eq.(5). Thus the complicated response of the factor R has weaker impact on estimation of these parameters than the direction of the source $\boldsymbol{\Omega}_s$ (see figure 2). Distance D or the orientation $\boldsymbol{\Omega}_l$ of the binary are determined by using the information of the amplitudes $\cos 2\psi_{AB} A_+$ and $\sin 2\psi_{AB} A_\times$. As seen in the expressions (B2)(B4)(B5)(B6)(B7), they are strongly correlated to the direction $\boldsymbol{\Omega}_s$ of the binary. Thus their estimation errors are also improved by the effect of the factor R.

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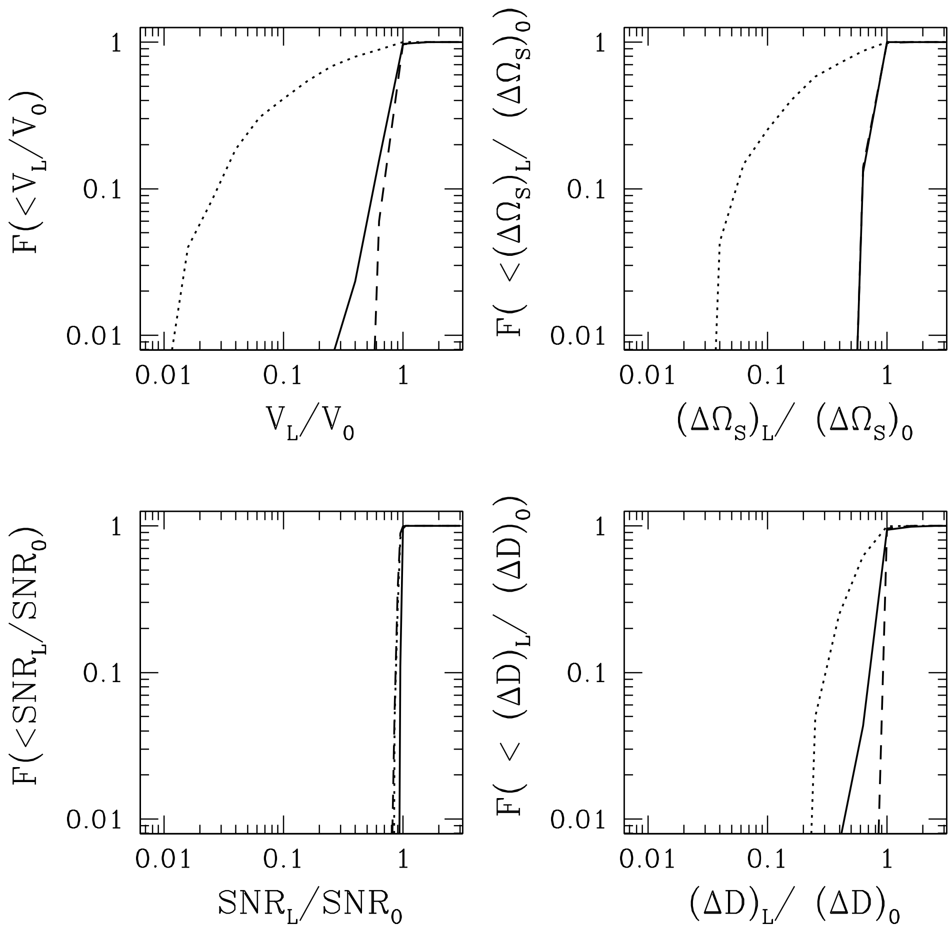

FIG. 1. Distribution of relative magnitude of the three dimensional error box ΔV, SNR, the angular resolution $\Delta\Omega_s$, and the distance error $\Delta D/D$. We compare results from two data streams (with suffix L) and those obtained by past simple method (with suffix 0). The solid lines are results for MBH binaries with redshifted masses $10^6+10^6M_\odot$, the doted lines for $10^5+10^5M_\odot$, and dashed lines for $10^4+10^4M_\odot$. We have analyzed 300 binaries with random directions and orientations.

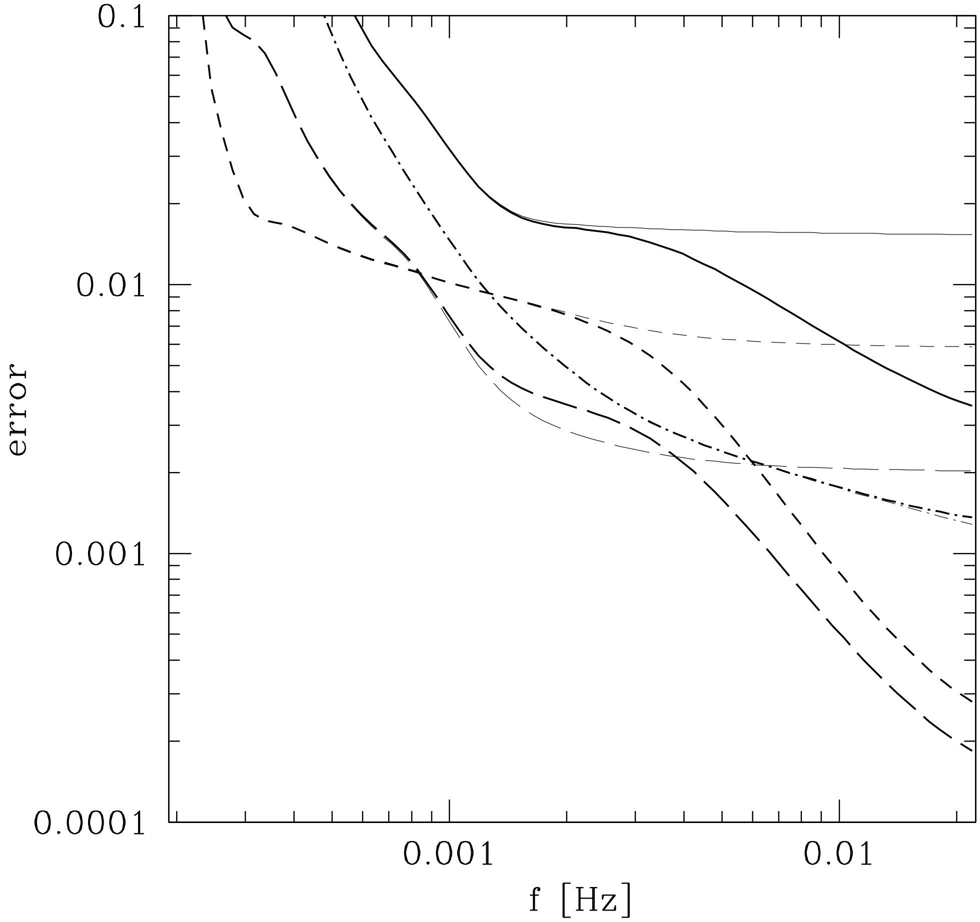

FIG. 2. Dependence of the parameter estimation errors $\Delta\gamma_i$ on the upper cut-off frequency $f_{cut} \leq f_{isco}$. We star integration of the signal from $f_0 = 1.9 \times 10^{-4}$Hz. The MBH binary has redshifted masses $10^5 + 10^5 M_\odot$, and exists at $z = 1$ with direction $\theta_s = 2.49, \phi_s = 0.03$ and orientation $\theta_s = 2.32, \phi_s = 4.46$. The thin lines are the past estimations and thick ones are the new estimations. The solid lines represent for $\Delta D/D$, the long dashed lines for $\Delta\Omega_l$, the short-dashed lines for $\Delta\Omega_s$, and the dash-dotted lines for the reduced mass $\Delta\mu_z/\mu_z$. SNR becomes $\sim$ 1054 for $f_{cut} = 7.0 \times 10^{-3}$Hz and $\sim$ 1140 for $f_{cut} = 2.0 \times 10^{-2}$Hz.

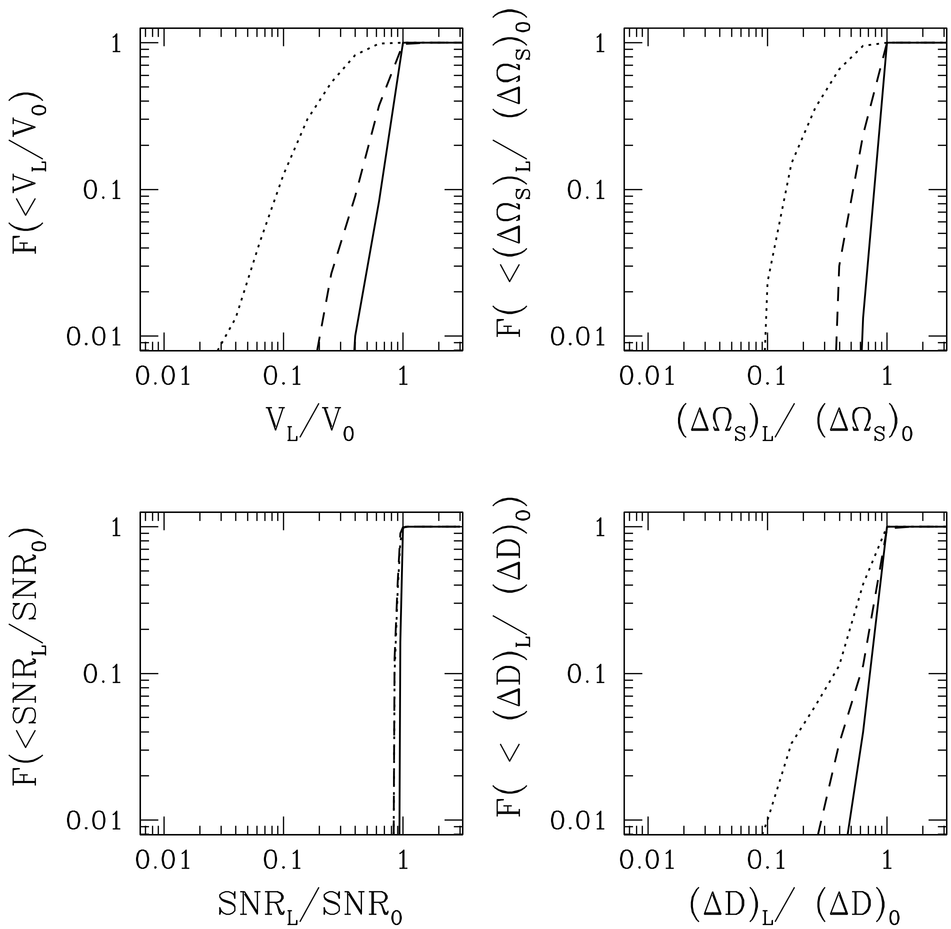

FIG. 3. Same as figure 1, but for the noise canceling combination X.